Application of Nonlinear Concrete Model for Finite Element Analysis of Reinforced Concrete Beams

V. B. Dawari, G. R. Vesmawala

Abstract—The non-linear behavior of reinforced concrete (RC) beams till the ultimate failure is a complicated phenomenon due to the involvement of heterogenic material properties and cracking behavior of concrete. Behavior prediction of reinforced concrete elements till failure is usually carried out using experimental testing, and the observations are recorded only at critical locations due to restriction in cost of testing equipment and accessories. In order to avoid the destructive testing, reduction of the cost of materials and manpower, the behavior prediction of RC beams is generally carried out using numerical methods. This paper presents study on non-linear flexural behavior of reinforced concrete beams. Non-linear finite element analysis of reinforced concrete beams under flexural loading is presented in this paper. Finite element modelling of reinforced concrete beams is carried out using discrete reinforcement modelling technique. The capability of the model to capture the critical crack regions, loads and deflections for various loadings in reinforced concrete beam has been illustrated. Comparison is made between the experimental results and finite element analyses with respect to initial crack formation and the ultimate load capacity of beams. The results obtained in the present study show close agreement with those in the available literature.

Index Terms—.ANSYS, Discrete model, Flexural cracks, Load-Displacement Curve, Nonlinear finite element analysis, Reinforced concrete, Ultimate load.

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1 INTRODUCTION

Reinforced concrete (RC) has become one of the most important building materials and is widely used in many types of engineering structures. The economy, the efficiency, the strength and the stiffness of reinforced concrete make it an attractive material for a wide range of structural applications. Understanding the response of the structural components during loading is crucial to the development of an overall efficient and safe structure. Different methods have been utilized to study the response of structural components. Typically, the behavior of reinforced concrete beams is studied by full-scale experimental investigations. The results are compared to theoretical calculations that estimate deflections and internal stress/strain distributions within the beams. Finite element analysis can also be used to model the behavior numerically to confirm these calculations, as well as to provide a valuable supplement to the laboratory investigations, particularly in parametric studies.

Buckhouse [1] studied the flexural behavior of RC beams experimentally and compared the results with analytical values. Wolanski [2] has verified and validated the experimental test results provided by Buckhouse [1] using the finite element software ANSYS. Kachlakev, et al. [3] used ANSYS to study concrete beam members with externally bonded Carbon Fiber Reinforced Polymer (CFRP) fabric. The study indicated that the use of a finite element program ANSYS to model experi-

mental data was viable and the results were reasonably well.

Fanning [4] used 3.0 m long ordinarily RC beams with cross section of 155 mm x 240 mm to validate the FE analysis using ANSYS. The results of the FE models were found to be particularly sensitive to the Young's modulus of the concrete and the yield strength of the reinforcement. Dahmani et al. [5] studied the crack propagation in RC beams using ANSYS software modelled with Solid65 element with smeared reinforcement approach. Vasudevan and Kothandaraman [6] conducted non-linear analysis of six numbers of RC beams using ANSYS based on IS 456: 2000 guide lines and compared the results with analytical calculations. Vasudevan and Kothandaraman [7, 8] explored the advantages of batch mode approach and non-linear FE analysis has been carried out by creating a single batch file from modelling to complete post processing.

In this paper, the nonlinear material model of concrete in commercially available finite element software ANSYS v12 is used to evaluate the ultimate load, Load-midspan displacement relationship and cracks development of reinforced concrete beams.

The validity of the model is calibrated against the results of previous experimental studies. The beam model used by Buckhouse [1] is considered for the first numerical example. This paper also compares the results of flexural response obtained using the discrete approach in the present study to the smeared approach used by Dahmani [5] in the second numerical example. The third numerical example demonstrates the ANSYS model to validate the experimental results.

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2 FINITE ELEMENT MODELLING

2.1 Element Types

Concrete: The Solid65 element is used to model the con-

crete. This element has eight nodes with three degrees of freedom at each node – translations in the nodal x, y, and z directions. This element is capable of plastic deformation, cracking in three orthogonal directions, and crushing. A schematic of the element is shown in Fig 1(a).

Steel Reinforcement: A Link8 element is used to model steel reinforcement. This element is a 3D spar element and it has two nodes with three degrees of freedom – translations in the nodal x, y, and z directions. This element is also capable of plastic deformation. This element is shown in Fig. 1(b).

al stress-strain relationship for the concrete is obtained using modified Hognestad piecewise elastic model [10]. Following equations are used to compute the multilinear isotropic stress-strain curve for the concrete (Fig. 3).

$$E_c = 57000 f_c^{'} \tag{1}$$

$$f = \frac{E_c \varepsilon}{1 + \left(\dfrac{\varepsilon}{\varepsilon_0}\right)^2} \tag{2}$$

$$\varepsilon_0 = \frac{2 f_c^{'}}{E_c} \tag{3}$$

$$E_c = \frac{f}{\varepsilon} \tag{4}$$

where:
f = stress at any strain ε, psi
ε = strain at stress f
ε_0 = strain at the ultimate compressive strength $f_c^{'}$

(a)

(b)

Fig. 1. *(a)* Solid 65 Element (b) Link 8 Element

Fig. 2. Typical uniaxial compressive and tensile stress-strain curve for concrete [9]

2.2 Material Properties

Concrete: Concrete is a quasi-brittle material and has highly nonlinear and ductile stress strain relationship. The nonlinear behavior attributed to the formation and gradual growth of micro cracks under loading. The tensile strength of concrete is typically 8-15% of the compressive strength. Fig. 2 shows a typical stress-strain curve for normal weight concrete [9]. In compression, the stress-strain curve for concrete is linearly elastic up to about 30% of the maximum compressive strength. Above this point, the stress increases gradually up to the maximum compressive strength. After it reaches the maximum compressive strength σ_{cu} , the curve descends into a softening region, and eventually crushing failure occurs at an ultimate strain ε_{cu}. In tension, the stress-strain curve for concrete is approximately linearly elastic up to the maximum tensile strength. After this point, the concrete cracks and the strength decreases gradually to zero [9].

Various mathematical models are available to approximate this nonlinear behavior. In this paper, the compressive uniaxi-
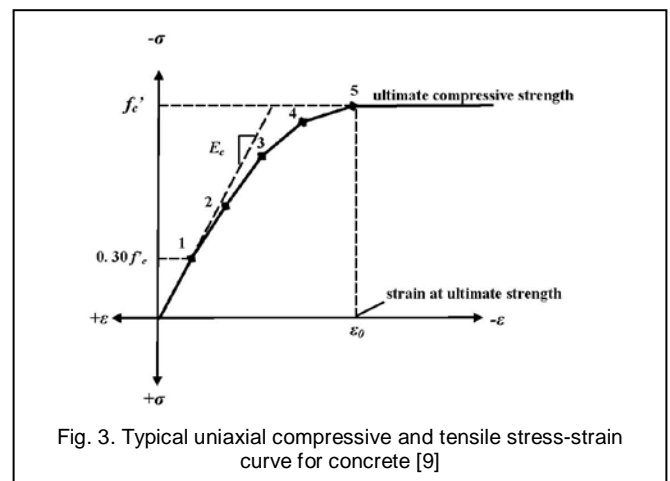

Fig. 3. Typical uniaxial compressive and tensile stress-strain curve for concrete [9]

The model is capable of predicting failure for concrete materials (Fig. 4). Both cracking and crushing failure modes are accounted for. The two input strength parameters i.e., ultimate uniaxial tensile and compressive strengths are needed to de-

fine a failure surface for the concrete. Consequently, a criterion for failure of the concrete due to a multiaxial stress state can be calculated [11].
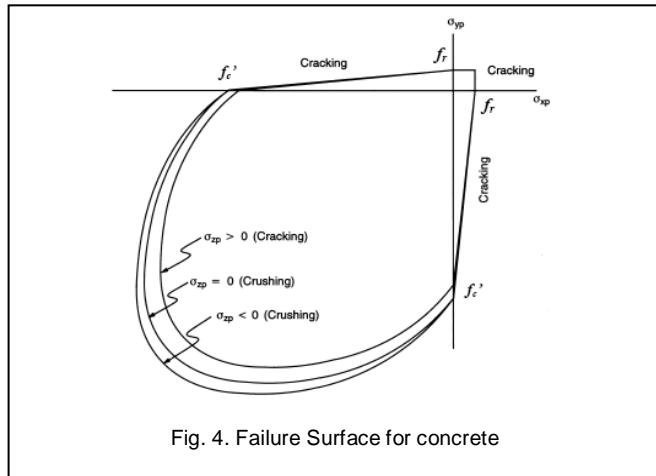

Fig. 4. Failure Surface for concrete

In a concrete element, cracking occurs when the principal tensile stress in any direction lies outside the failure surface. After cracking, the elastic modulus of the concrete element is set to zero in the direction parallel to the principal tensile stress direction. Crushing occurs when all principal stresses are compressive and lies outside the failure surface; subsequently, the elastic modulus is set to zero in all directions, and the element effectively disappears. During the study of past researchers [2],[3], it was found that if the crushing capability of the concrete is turned on, the finite element beam models fail prematurely. Therefore, in this study, the crushing capability was turned off and cracking of the concrete controlled the failure of the finite element models.

For Implementation of the Willam and Warnke [11], material model in ANSYS, nine constants are defined as : Shear transfer coefficients for an open crack; Shear transfer coefficients for a closed crack; Uniaxial tensile cracking stress; Uniaxial crushing stress (positive); Biaxial crushing stress (positive); Ambient hydrostatic stress state for use with constants 7 and 8; Biaxial crushing stress (positive) under the ambient hydrostatic stress state (constant 6); Uniaxial crushing stress (positive) under the ambient hydrostatic stress state (constant 6); Stiffness multiplier for cracked tensile condition.

Steel Reinforcement: The mechanical behavior of reinforcing steel bar is assumed to be elastic bilinear under monotonic tension. The steel bar initially exhibits linear elastic portion followed by a yield platue, strain hardening and then stress drops till fracture occurs. The behavior of steel bar is same in compression and tension loading, as shown in Fig. 5. For incorporating steel material model, the essential inputs are modulus of elasticity, tangent modulus and the yield strength.
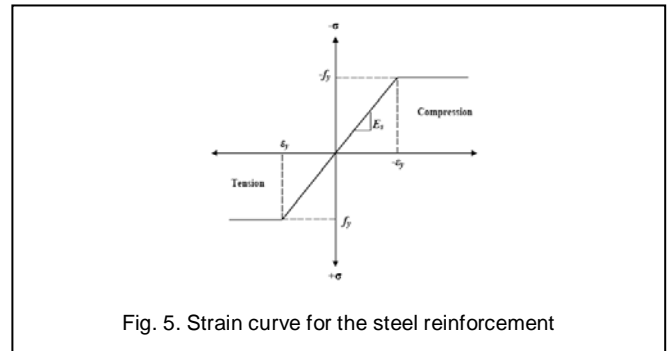

Fig. 5. Strain curve for the steel reinforcement

2.3 Finite Element Modelling of Steel Reinforcement

There are three techniques that exist to model steel reinforcement in finite element models for reinforced concrete: the discrete model, the embedded model, and the smeared model [12]. In the work presented in this paper, discrete modelling technique is used for modelling the reinforcement. The reinforcement in the discrete model (Fig. 6) uses bar or beam elements that are connected to concrete mesh nodes. Therefore, the concrete and the reinforcement mesh share the same nodes and concrete occupies the same regions occupied by the reinforcement.
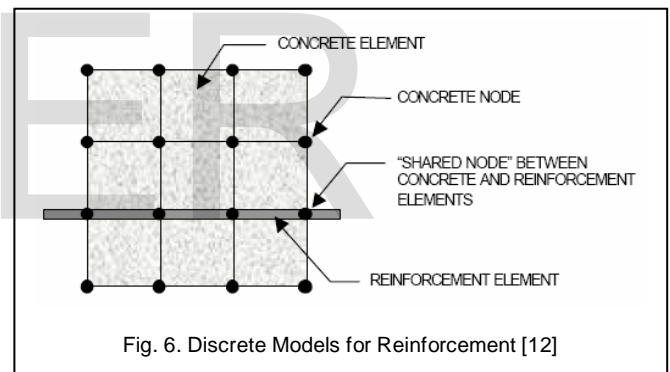

Fig. 6. Discrete Models for Reinforcement [12]

3 ANALYSIS OF RC BEAMS USING ANSYS

3.1 Modelling and Meshing

The beam is modelled by creating volumes. To obtain good results from the Solid65 element, rectangular mesh is used. The individual elements of reinforcement are created in the modelling through the nodes created by the mesh of the concrete volume. The command 'merge items' merges separate entities that have the same location. These items will then be merged into single entities.

Displacement boundary conditions are needed to constrain the model to get a unique solution. To ensure that the model acts the same way as the experimental beam, boundary conditions need to be applied at the supports and loadings. The support is modelled in such a way that a hinge and roller is created. The force applied is applied across the entire modal line.

3.2 Non Linear Analysis

In nonlinear analysis, the total load applied to a finite element model is divided into a series of load increments called load steps. At the completion of each incremental solution, the stiffness matrix of the model is adjusted to reflect nonlinear changes in structural stiffness before proceeding to the next load increment. The ANSYS program uses Newton-Raphson equilibrium iterations for updating the model stiffness. Newton-Raphson equilibrium iterations provide convergence at the end of each load increment within tolerance limits.

3.3 Load Stepping and Failure Definition for FE Models

For the nonlinear analysis, automatic time stepping in the ANSYS program predicts and controls load step sizes. Based on the previous solution history and the physics of the models, if the convergence behavior is smooth, automatic time stepping increases the load increment up to a selected maximum load step size. The maximum and minimum load step sizes are required inputs for the automatic time stepping.

Failure for each of the models is defined when the solution for a 1 kN load increment still does not converge. The program then gives a message specifying that the models have a significantly large deflection, exceeding the displacement limitation of the ANSYS program.

4   NUMERICAL EXAMPLE 1

4.1 Problem Statement

For the validation of the modelling and analysis techniques using ANSYS 12, the RC beam model used by Buckhouse [1] is considered. For flexural analysis, a RC beam is modelled in ANSYS 12 using GUI with the details given by Buckhouse [1]. The geometry of the beam is shown in Fig. 7. The FE model in ANSYS is shown in Fig. 8. The properties of steel and concrete are given in Table 1.
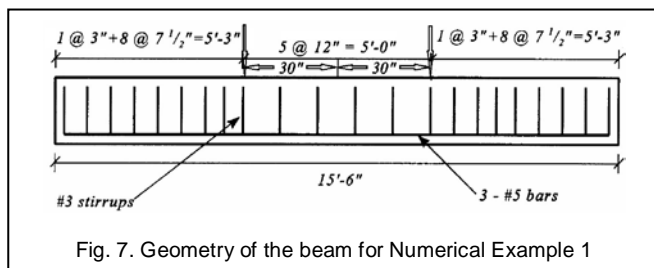

Fig. 7. Geometry of the beam for Numerical Example 1

TABLE 1
PROPERTIES FOR STEEL AND CONCRETE

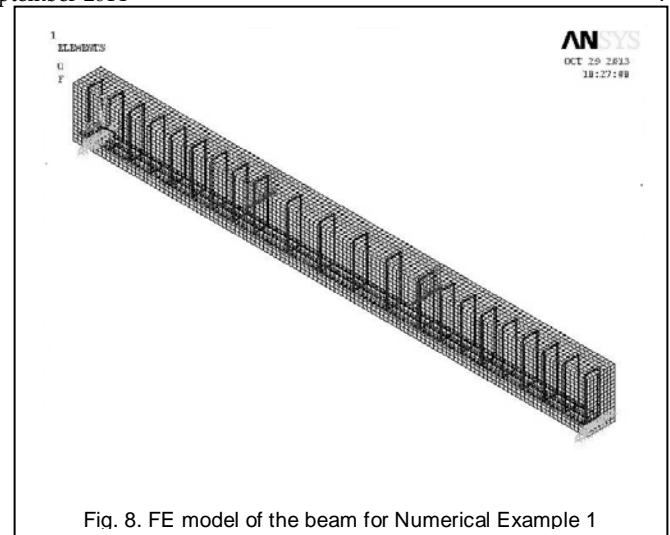| Property	Value
28-Day Compressive Strength of Concrete (psi)	4770
Modulus of Elasticity for concrete (psi)	3949076
Yield Stress of Steel (psi)	60000
Modulus of Elasticity for steel (psi)	29000000
Poisson's ratio	0.3

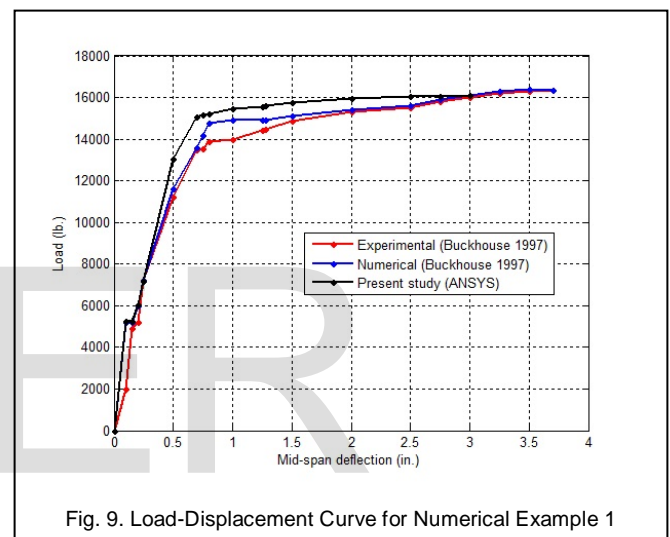Fig. 8. FE model of the beam for Numerical Example 1

Fig. 9. Load-Displacement Curve for Numerical Example 1

Left support is modelled as hinged and the right support is modelled as roller by constraining the displacement in the relevant directions. Load is given in increments (at each time step increment of 5000 lb).

4.2 Results and Discussion

TABLE 2
COMPARISON OF THE RESULTS OF FLEXURAL ANALYSIS

Beam	Ultimate Load (lb.)	Centre-line deflection (in.)
Present study (ANSYS)	16094	3.056
Experimental Study [1]	16310	3.650
Numerical Study [1][2]	16310	3.586

Fig. 9 shows the load vs. mid-span deflection plot of the present study. The Table 2 shows comparison of load and centre-line deflection at failure of beam. The results are compared with those reported by Buckhouse [1]. The load – deflection behavior and the values of Centreline deflections of the of the finite element model compare well to experimental data obtained from a reinforced concrete beam [1]. The failure mech-

anism of a reinforced concrete beam is modelled quite well using FEA, and the failure load predicted is quite close to the failure load measured during experimental testing [1]. The tri-linear behavior is observed from the load vs. mid-span deflection curve (Fig. 9) which is closely matching with results available in literature [1].

5 NUMERICAL EXAMPLE 2

5.1 Problem Statement

The beam model used by Dahmani et al. [5] is considered for the study. Three-dimensional nonlinear finite element model of a reinforced concrete beam has been developed by Dahmani et al. [5]. The beam specimen used is of size 3000 mm * 500 mm * 200 mm. Tension reinforcement of 2 bars of 16 mm diameter is used. Concrete of 28 days cube strength is 27 MPa and reinforcing steel is with yield stress 525 MPa and clear cover of 25 mm is considered in this study. A central point load is applied for the model. SOLID65 solid elements are used for modelling, where the compressive crushing of concrete is facilitated using plasticity algorithm while the concrete cracking in tension zone was accommodated by the non-linear material model. Smeared reinforcement technique was adopted by Dahmani et al. [5] and reinforcement was introduced as a percentage of steel embedded in concrete. The capability of the model to capture the critical crack regions, loads and deflections for various types of loadings in reinforced concrete beam has been illustrated.
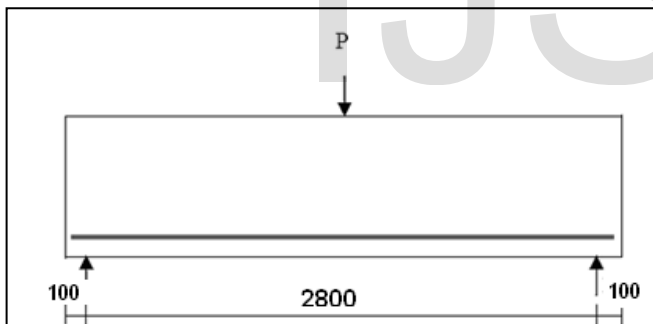

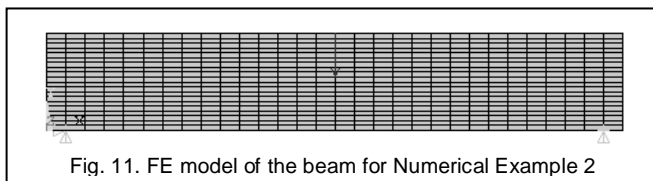Fig. 10. Geometry of the beam for Numerical Example 2

Fig. 11. FE model of the beam for Numerical Example 2

In the present study, the same geometry and the material properties are used with discrete reinforcement modelling method and the results are compared with that of literature [5]. Fig. 10 and Fig. 11 show the geometry and FE model of the beam considered for the study.
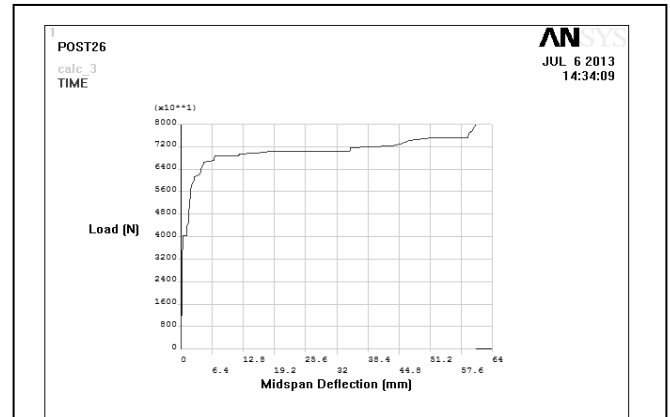

Fig.12. Load-Displacement Curve obtained in the present study for Numerical Example 2
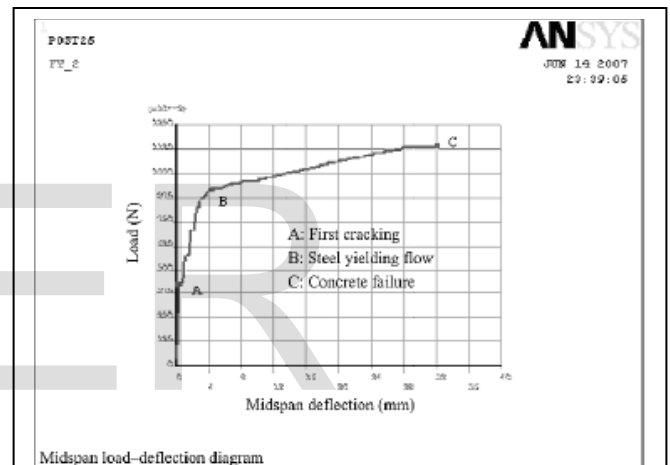

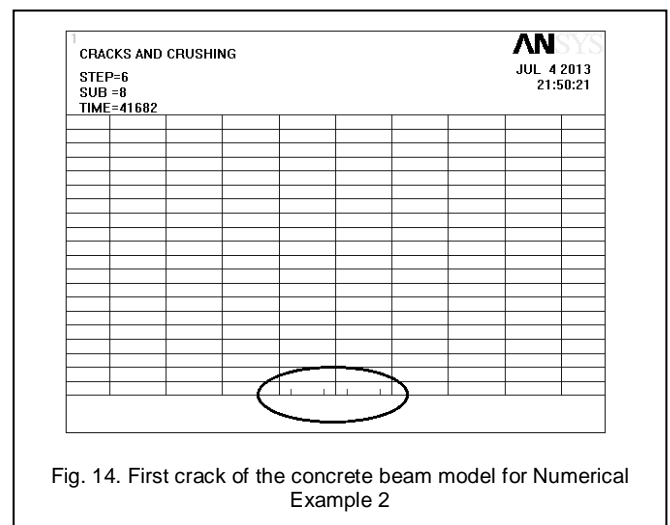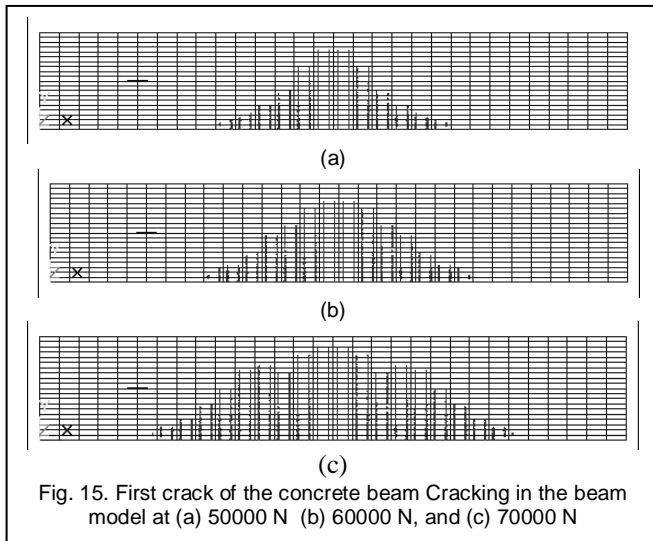Fig.13. Load-Displacement Curve obtained by Dahmani et al. [5]

Fig. 14. First crack of the concrete beam model for Numerical Example 2

Fig. 15. First crack of the concrete beam Cracking in the beam model at (a) 50000 N (b) 60000 N, and (c) 70000 N

5.2 Results and Discussion

Load - displacement curves are plotted as shown in Fig. 12 show the close agreement with the available literature (Dahmani et al. [5]) in the elastic region as shown in Fig. 13. The first crack load as shown in Fig. 14, i.e. 41682 N is consistent with a load of 42602 N (Dahmani et al. [5]). In the non-linear region of the response, subsequent cracking occurs as more loads are applied to the beam. Cracks grow in the constant moment region, and the beam starts cracking out towards the supports at a load of 50,000 N (Fig. 15(a)). Significant flexural cracking occurs in the beam at 60,000 N (Fig. 15 (b)). Also, diagonal tension cracks start to form at load of 70,000 N (Fig. 15 (c)). Yielding of steel reinforcement occurs when a force of 68748 N unlike 94,000 N (Dahmani et al. [5]),) is applied. At this point the displacements of the beam begin to increase at a higher rate as more loads are applied.
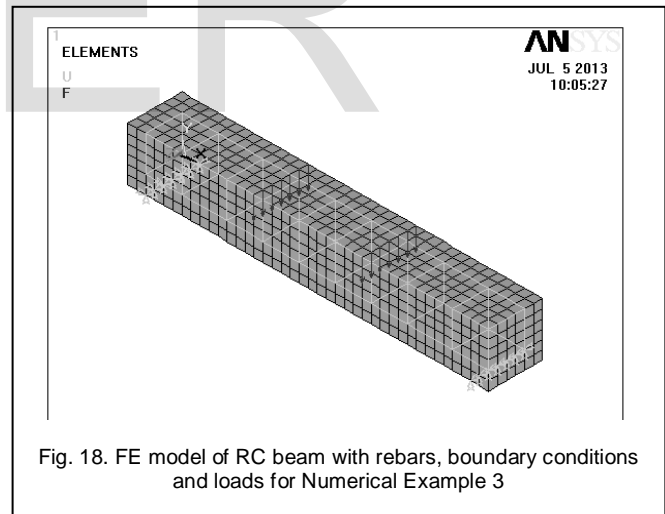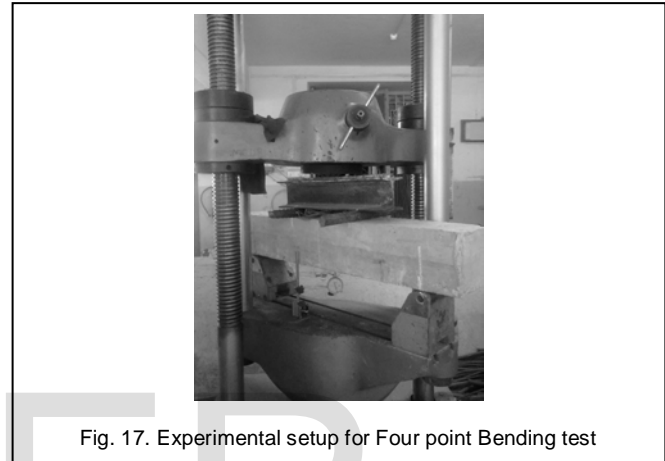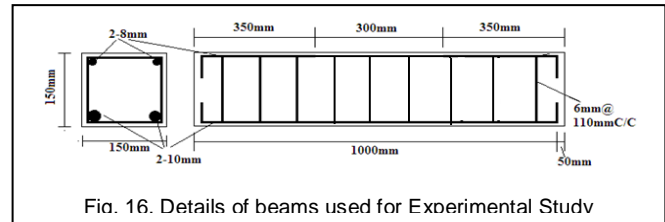
6 NUMERICAL EXAMPLE 3

6.1 Problem Statement

For the Finite element and experimental analysis, three beam specimen of size 1000 mm * 150 mm * 150 mm are considered and details are shown in Fig. 16. Tension bars are of 2 numbers 10 mm diameter, hanger bars are 2 numbers 8 mm diameter and shear reinforcement of two legged 6 mm at 110 mm spacing is used in this study. Concrete of 28 days cube strength as 27 MPa and reinforcing steel with yield stress 525 MPa is used. The clear cover of 25 mm is used in experimental testing. These beams were tested and analysed for four point bending test under loading with distance of 350 mm from either end. The material properties were derived from IS 456:2000 codal provisions and used for plotting multilinear stress strain curve for concrete.

Flexure testing of the beams is carried out using a Universal Testing Machine of capacity 600kN The two point loading is applied through a spreader beam with a constant moment zone of 300 mm. The clear span of 900 mm is set for testing. Testing arrangements are shown in Fig. 17. The deflections are

recorded at midspan using dial gauge of least count 0.01 mm. Hinge and roller support condition is modelled in FE model with span of 900 mm by restraining linear displacements. Fig. 18 shows details of FE model.

Fig. 16. Details of beams used for Experimental Study

Fig. 17. Experimental setup for Four point Bending test

Fig. 18. FE model of RC beam with rebars, boundary conditions and loads for Numerical Example 3

6.2 Results and Discussion

Results and Discussions: Plots of load versus mid-span deflection from FEA and experiments and are shown in Fig. 19 for beams. The shapes of the load–deflection plots are trilinear. Immediately after the first crack formation, there is a small kink in the FEA plot, which is due to sudden loss of moment of inertia after the first crack formation. The comparison of experimental and FEA curves indicated that the experimental and FEA results are in good agreement. The values for ultimate load show consistency in FE and experimental results. Figure shows the nodal deformation and cracking regions

generated by FEA. Fig. 20 and Fig. 21show cracks at initial cracking load and ultimate load.
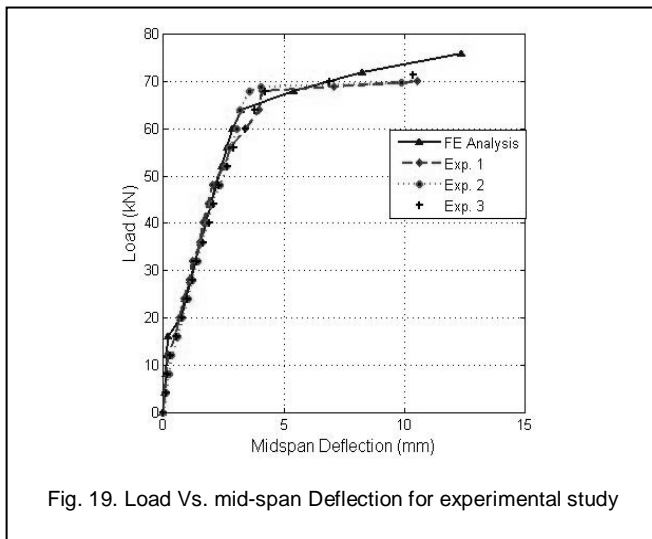

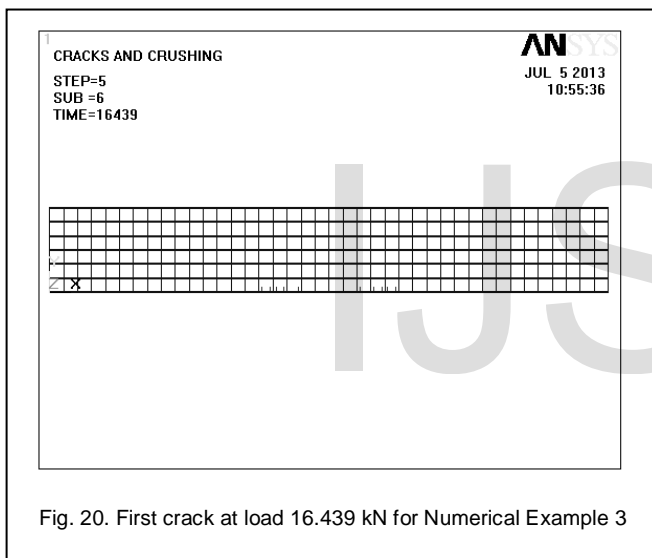Fig. 19. Load Vs. mid-span Deflection for experimental study

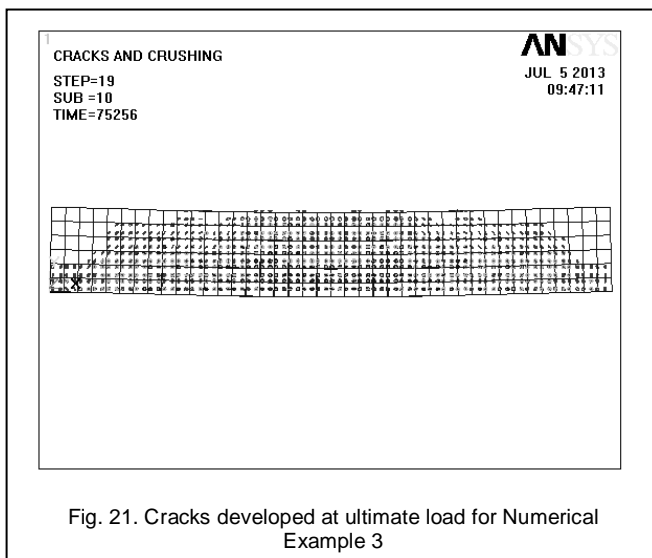Fig. 20. First crack at load 16.439 kN for Numerical Example 3

Fig. 21. Cracks developed at ultimate load for Numerical Example 3

7 CONCLUSIONS

Finite element models of reinforced concrete beams, constructed in ANSYS 12 using the dedicated concrete elements have accurately captured the nonlinear flexural response of these systems up to failure. The failure mechanism of a reinforced concrete beam is modelled quite well using FEA, and the failure load predicted is very close to the failure load measured during experimental testing. The analysis procedure used in this paper and various output plots constructed by FEA have provided a deeper insight for future application of finite element software for the non-linear analysis of RC beams. Based on the analyses carried out on the RC beams using ANSYS, it is found that results are more sensitive to mesh size, materials properties, load increments, etc.

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